Name: ____

Summer Assignment

IB Math Studies

Welcome to IB Math Studies! I am looking forward to a great school year. Please complete this packet of Geometry and Algebra II review problems. This will help you keep your skills sharp so that you are ready for the start of school in September. You should have this packet complete for the first *full* day of school. **Bring the packet to class on the first *full* day of school**.

This assignment will be checked for ***completeness***. All problems should be done or well attempted. Show work on every problem in the space provided. Please write neatly.

You will be assessed on the topics presented in this packet during the first few weeks of school. You will be given an opportunity to ask questions in class in the days prior to the assessment, but if you have **<u>significant</u>** trouble completing this packet you should contact your guidance counselor to reconsider your course placement or speak with your teacher about extra help opportunities.

These topics will be on the IB exam, but will not be covered extensively in class since they have been covered in Algebra I, Geometry and Algebra II. The first several pages of this packet contain review material and example problems. Use these examples to help you complete the assignment.

Enjoy your summer!
Mrs. Manning

<u>Calculator requirement</u>: **A graphing calculator is required for this course. It is suggested that you purchase a TI-84/84+ graphing calculator before you come to school in September! A TI-83/83+ is also acceptable, but lacks some nice features that the TI-84 has. A TI-89 is not acceptable and will not be allowed on the IB Exam. Look for sales in the summer (Wal-Mart, Target, Staples, etc). Please email your teacher with any questions.**

Summary Packet Review Material

Coordinate Geometry

SECTION I: *Distance and Midpoint Formulae*

Given two points (x_1, y_1) and (x_2, y_2):

Distance between the points = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of the points = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Examples:

a) Find the value(s) of *b* given that A(3, -2) and B(*b*, 1) are $\sqrt{13}$ units apart.

$$\sqrt{13} = \sqrt{(b-3)^2 + (1-(-2))^2}$$
$$13 = (b-3)^2 + 3^2$$
$$4 = (b-3)^2$$
$$\sqrt{4} = \sqrt{(b-3)^2}$$
$$\pm 2 = b - 3$$
$$3 \pm 2 = b$$
$$b = 5,1$$

b) Find the coordinates of B if M is the midpoint of AB, A is at (1, 3) and M is at (4, -2).

$$\left(\frac{1+x}{2}, \frac{3+y}{2}\right) = (4,-2)$$
$$\frac{1+x}{2} = 4 \qquad \frac{3+y}{2} = -2$$
$$1 + x = 8 \qquad 3 + y = -4$$
$$x = 7 \qquad y = -7$$

SECTION II: *Gradient, Equations of Lines and Graphing*

Vocabulary and Information:

- *Gradient* is another word for slope. Formula for slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$
- The gradient of a horizontal line is zero. Its equation is of the form *x* = *a* where *a* is a constant.
- The gradient of a vertical line is undefined. Its equation is of the form *y* = *b* where *b* is a constant.
- Parallel lines have the same gradient. Perpendicular lines have opposite reciprocal gradients.
- Three or more points are *collinear* if they lie on the same line.
- Two forms for writing equations of lines:
  - Gradient-Intercept Form: *y* = m*x* + b, where b is the *y*-intercept and m is the slope
  - General Form: A*x* + B*y* = C, where A, B, and C are integers. The gradient of a line in this form can be found by calculating $-\frac{A}{B}$.
- Axis Intercepts: To find an *x*-intercept, set *y* equal to zero and solve for *x*. To find a *y*-intercept, set *x* equal to zero and solve for *y*.

Examples:

a) Find t given that the line joining D(-1, -3) to C(1, t) is perpendicular to a line with gradient 2.

Line DC has slope – ½ (opposite reciprocal of 2).

$$\frac{t+3}{1+1} = -\frac{1}{2}$$

$$\frac{t+3}{2} = -\frac{1}{2}$$

$$t+3 = -1$$

$$t = -4$$

b) Find the equation of the line, in both gradient-intercept form and general form, passing through (5, 3) and (1, 1).

$$m = \frac{3-1}{5-1} = \frac{2}{4} = \frac{1}{2}$$

$$y = \frac{1}{2}x + b$$

$$1 = \frac{1}{2}(1) + b$$

$$\frac{1}{2} = b$$

Gradient-intercept form: $y = \frac{1}{2}x + \frac{1}{2}$

$$-\frac{1}{2}x + y = \frac{1}{2}$$

$$2\left(-\frac{1}{2}x + y = \frac{1}{2}\right)$$

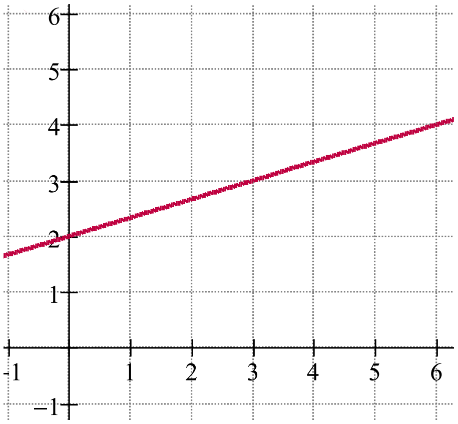
$$-x + 2y = 1$$

General Form

c) Graph the line $y = \frac{1}{3}x + 2$.

y-intercept (0, 2)

slope = 1/3



d) Graph the line 2x – 3y = 12 using axis-intercepts.

Set x = 0: – 3y = 12

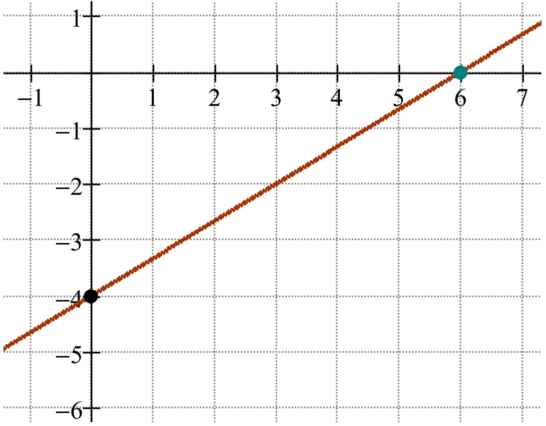
y = -4

Point: (0, -4)

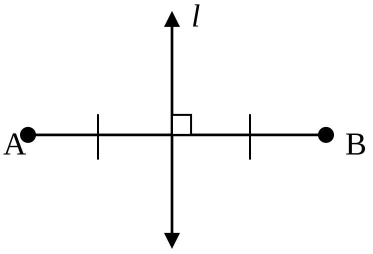
Set y = 0: 2x = 12

x = 6

Point (6, 0)



SECTION III: *Perpendicular Bisectors*



Line l is the perpendicular bisector of segment AB. Since l bisects segment AB, we know that the midpoint of AB is on the perpendicular bisector. Also, points on the perpendicular bisector are equidistant from points A and B.

Example: Find the equation of the perpendicular bisector of AB for A(-1, 2) and B(3, 4).

Find the midpoint of AB: $\left(\frac{-1+3}{2},\frac{2+4}{2}\right)=(1,3)$ This is a point on the perpendicular bisector.

Find the slope of AB: $m=\frac{4-2}{3+1}=\frac{1}{2}$. Therefore, the slope of the *perpendicular* bisector is -2.

Use the slope of the perpendicular bisector (-2) and a point on the perpendicular bisector (1, 3) to write the equation.

$$y=-2x+b$$
$$3=-2(1)+b$$
$$5=b$$
$$y=-2x+5$$

Quadratic Algebra

SECTION IV: *Expansion Rules*

FOIL: $(a+b)(c+d)=ac+ad+bc+bd$

Difference of Two Squares: $(a+b)(a-b)=a^2-ab+ab-b^2=a^2-b^2$

Perfect Squares Expansion:
$$(a+b)^2=(a+b)(a+b)=a^2+2ab+b^2$$
$$(a-b)^2=(a-b)(a-b)=a^2-2ab+b^2$$

Further Expansion: $(a+b)(c+d+e)=(a+b)c+(a+b)d+(a+b)e$
$$=ac+bc+ad+bd+ae+be$$

SECTION V: *Factorization of Quadratic Expressions*

A **quadratic expression** in *x* is an expression of the form ax^2+bx+c where *x* is the variable and *a, b* and *c* represent constants with $a \neq 0$. **Factorization** is the reverse process of expansion.

Example 1: Removal of common factors. (GCF—Greatest Common Factor)

a)
$$2x^2-6x$$
$$2x(x-3)$$

b)
$$(x+2)^2+2x+4$$
$$(x+2)^2+2(x+2)$$
$$(x+2)\left[(x+2)+2\right]$$
$$(x+2)(x+4)$$

Example 2: Difference of Two Squares. $a^2-b^2=(a+b)(a-b)$

The sum of two squares is *prime* (not factorable).

a)
$$4x^2-25$$
$$(2x-5)(2x+5)$$

b)
$$(3x^2-48)$$
$$3(x^2-16)$$
$$3(x-4)(x+4)$$

Example 3: Quadratic Trinomial Factorization ($ax^2 + bx + c$ when $a = 1$). Find two values that multiply to c and add to b.

$$(x+p)(x+q) = x^2 + qx + px + pq = x^2 + (p+q)x + pq$$

$$\text{Therefore, } x^2 + (p+q)x + pq = (x+p)(x+q)$$

a) $x^2 + 5x + 4$
$(x+1)(x+4)$

b) $x^2 - x - 12$
$(x-4)(x+3)$

c) $3x^2 - 9x + 6$
$3(x^2 - 3x + 2)$
$3(x-2)(x-1)$

Example 4: Factoring $ax^2 + bx + c$ $(a \neq 1)$

Trial and Error
Use FOIL to help you guess and check:
Firsts = ax^2
Outers + Inners = bx
Lasts = c

Grouping Method (‘splitting the x-term’)
Find the product ac and then the factors of ac which add to b.
Split the middle term into the two factors.
Group in pairs and factor each pair.
Complete the factorization.

a) $3x^2 + 13x + 4$

By trial and error:
Firsts = $3x^2$, Lasts = 4
Outers + Inners = $13x$
Guess: $(3x+1)(x+4)$
Check outers and inners:
$12x + 1x = 13x$ ☺

By grouping:
Product = ac = 12
Find two factors of 12 that add to b = 13
Factors: 12 and 1
Rewrite the expression by splitting the middle term:
$3x^2 + 12x + 1x + 4$
Group and factor:
$(3x^2 + 12x) + (1x + 4)$
$3x(x+4) + 1(x+4)$
$(3x+1)(x+4)$

In general, look for the greatest common factor first. Then determine how the method to use by looking at the number of terms in the expression.

SECTION VI: *Solving Quadratic Equations*

A **quadratic equation** in x is an equation of the form $ax^2 + bx + c = 0$ where x is the unknown and a, b, and c are constants with $a \neq 0$.

Example 1: Solve for x by isolating the quadratic variable or expression and taking the square root of both sides.

a) $2x^2 - 1 = 31$
$2x^2 = 32$
$x^2 = 16$
$x = \pm 4$

b) $(x-3)^2 = 16$
$x - 3 = \pm 4$
$x = 3 \pm 4$
$x = 7, -1$

c) $(x+2)^2 = 11$
$x + 2 = \pm\sqrt{11}$
$x = -2 \pm \sqrt{11}$

d) $2 - 3x^2 = 8$
$-3x^2 = 6$
$x^2 = -2$
$\varnothing$

Example 2: Solving by using the zero-product property.
When the product of two or more numbers is zero, then at least one of them must be zero.

If *ab* = 0, then *a* = 0 or *b* = 0.

Set the equation equal to zero by moving everything to one side. Factor and apply the zero product property.

a)
$$x^2+3x=28$$
$$x^2+3x-28=0$$
$$(x+7)(x-4)=0$$
$$x+7=0 \qquad x-4=0$$
$$x=-7, \qquad x=4$$

b)
$$5x^2=3x+2$$
$$5x^2-3x-2=0$$
$$(5x+2)(x-1)=0$$
$$5x+2=0 \quad x-1=0$$
$$x=-\frac{2}{5}, \quad x=1$$

c)
$$2x(x-6)=x-20$$
$$2x^2-12x=x-20$$
$$2x^2-13x+20=0$$
$$(2x-5)(x-4)=0$$
$$x=-\frac{5}{2}, \quad x=4$$

d)
$$\frac{x-1}{x}=\frac{x+11}{5}$$
$$5(x-1)=x(x+11)$$
$$5x-5=x^2+11x$$
$$0=x^2+6x+5$$
$$0=(x+5)(x+1)$$
$$x=-5, \quad x=-1$$

Example 3: Solving by using the quadratic formula.
$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

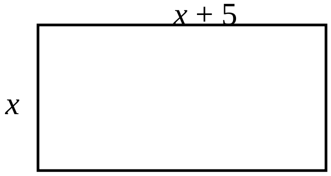
a)
$$4x^2-8x+1=0$$
$$x=\frac{8\pm\sqrt{64-4(4)(1)}}{2(4)}$$
$$x=\frac{8\pm\sqrt{64-16}}{8}$$
$$x=\frac{8\pm\sqrt{48}}{8}$$
$$x=\frac{8\pm4\sqrt{3}}{8}$$
$$x=\frac{2\pm\sqrt{3}}{2}$$

Example 4: Problem-solving with quadratics.

a) The product of a number and the number increased by 4 is 96. Find the two possible answers for the number.
$$x(x+4)=96$$
$$x^2+4x-96=0$$
$$(x+12)(x-8)=0$$
$$x=-12, \quad 8$$

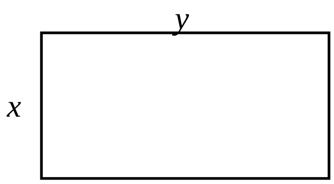
b) A rectangle has length 5 cm greater than its width. If it has an area of 84 cm^2, find the dimensions of the rectangle.
$$x(x+5)=84$$
$$x^2+5x-84=0$$
$$(x+12)(x-7)=0$$
$$x=-\cancel{12}, \quad 7$$

Dimensions: 7 by 12



c) A rectangular enclosure is made from 40 m of fencing. The area enclosed is 96 m^2. Find the dimensions of the enclosure.
$$\begin{cases} 2x+2y=40 & \rightarrow \quad x+y=20 \\ xy=96 & \qquad\quad y=20-x \end{cases}$$
$$x(20-x)=96$$
$$20x-x^2=96$$
$$0=x^2-20x-96$$
$$0=(x-12)(x-8)$$
$$x=12,8$$
$$y=8,12$$

Dimensions: 12 by 8



Practice Problems: Don't forget to show all work!!

SECTION I: Distance and Midpoint

1. Find b given that $P(b, 2)$ is equidistant from $A(5, 4)$ and $B(2, 5)$.

2. PQ is the diameter of a circle, center $(5, -\frac{1}{2})$. Find the coordinates of P given that Q is $(-1, 2)$.

SECTION II: Gradient, Equations of lines, Graphing

1. Find t given that the line joining $P(t, -2)$ to $Q(5, t)$ is perpendicular to a line with gradient $-1/3$.

2. Given the points $A(1, 3)$, $B(-1, 0)$, $C(6, 4)$, and $D(t, -1)$, find t if AC is parallel to DB.

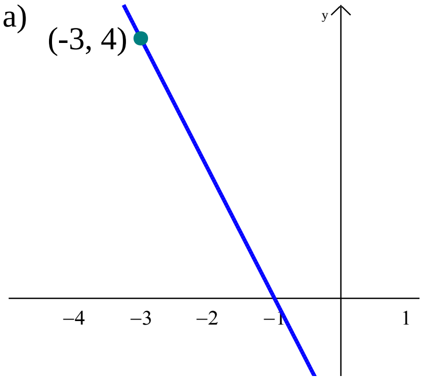
3. Find the equation of a line going through the point $(-1, 3)$ that is
   a) horizontal b) vertical

4. Find the equation of the line in *general form* which passes through the points $(5, -1)$ and $(-1, -2)$.

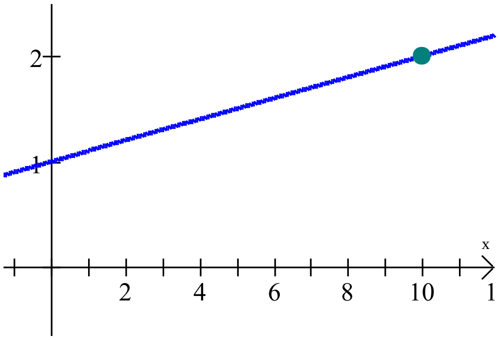
5. Find k if $(-1, 3)$ lies on the line with equation $5x - 2y = k$.

6. Write the equation in gradient-intercept form.

a)

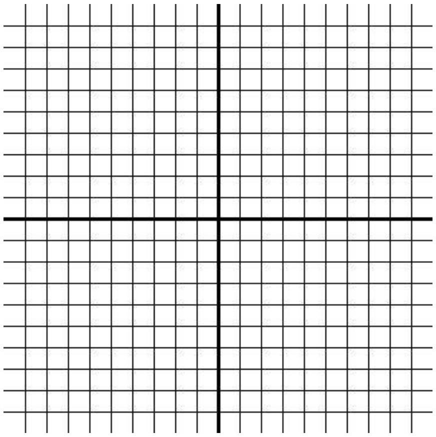


b)

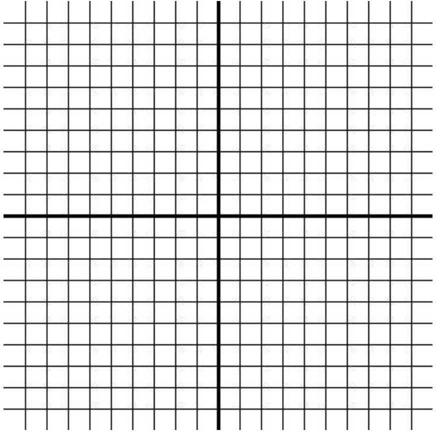


7. Draw the graph of the line given.

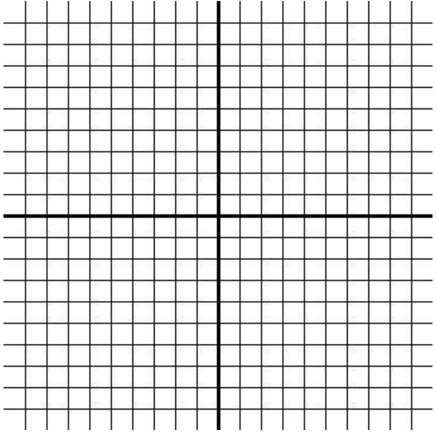
a) $y = 2x + 1$



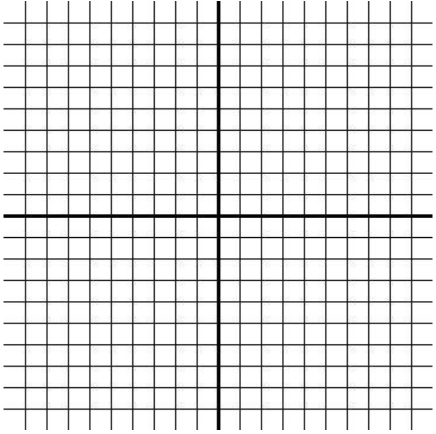
b) $y = \frac{2}{3}x + 2$



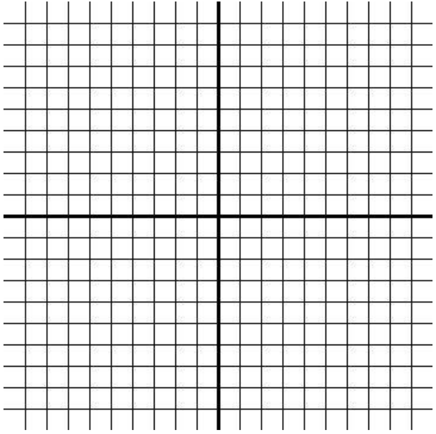
c) $4x + 3y = 12$



d) $9x - 2y = 9$



8. Graph the lines $x + y = 6$ and $2x - y = 6$ and find their point of intersection.



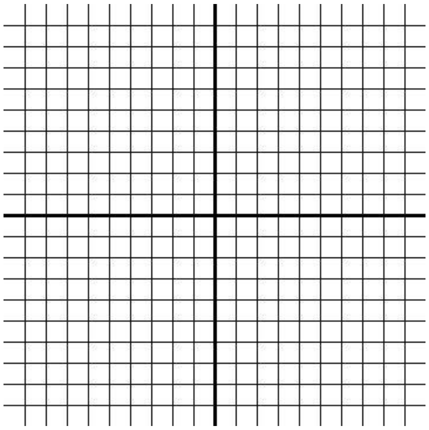
SECTION III: Perpendicular Bisectors

1. Find the equation of the perpendicular bisector of AB for A(3, 1) and B(-3, 6). (*Hint:* Find a point on the perpendicular bisector and the slope of the perpendicular bisector and write the equation of the line. Use the example in section III to help you.)

2. Segment PQ has endpoints P(7, -1) and Q(*a*, *b*). The segment is perpendicularly bisected by line *l* at the point (2, 2). (Use the graph to help you visualize this problem.)

   a. Find the values of *a* and *b*.

   b. Find the equation of the perpendicular bisector (line *l*).



SECTION IV: Expand and simplify.

1. $x(2x-1)-2x(5-x)$

2. $(2x+1)(3x-2)$

3. $(3x+4y)(3x-4y)$

4. $5-(x+2)^2$

5. $(2x+3)(x^2+4x+5)$

6. $(x+2)^3$

SECTION V: Use the method of your choice to factor the expression. If the expression is not factorable, write “prime”.

1. $x^2 + 10x + 25$

2. $3x^2 + 9x$

3. $x^2 + 4$

4. $4x^2 - 1$

5. $4x^2 - 8x - 60$

6. $x^2 + 3x - 40$

7. $2x^2 - 32$

8. $15x^2 + x - 2$

9. $3x^2 + 6x - 72$

10. $7x^2 + 21x - 28$

11. $6x^2 + 5x + 1$

12. $8x^2 + 14x + 3$

SECTION VI: Solve the quadratic equation using the given method.
Solve using square roots.

1. $1-3x^2=-8$

2. $\frac{1}{3}(2x-1)^2=8$

Solve using the zero-product property.

3. $10-3x=x^2$

4. $3x^2+8x=3$

5. $4x^2=5x$

6. $10x^2=7x+3$

7. $3x(x+2)-5(x-3)=17$

8. $\frac{2x}{3x+1}=\frac{1}{x+2}$

Solve by using the quadratic formula.

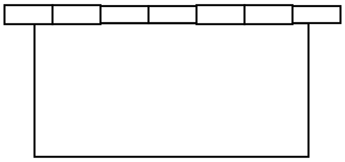
9. $3x^2+2x-2=0$

Use problem-solving strategies to solve.

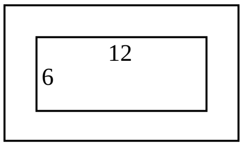
10. When 24 is subtracted from the square of a number, the result is five times the original number. Find the number.

11. A triangle has base 2 cm more than its altitude (height). If its area is 49.5 cm^2, find its altitude.

12. A rectangular pig pen is built against an existing brick fence. 24 m of fencing was used to enclose 70 m^2. Find the dimensions of the pen. (*Hint:* Write an equation for area and an equation for perimeter and solve the system of equations.)



13. A rectangular swimming pool is 12 m long by 6 m wide. It is surrounded by pavement of uniform width, the area of the pavement being $\frac{7}{8}$ the area of the pool.



a. If the pavement is x m wide, show that the area of the pavement is $4x^2 + 36x \text{ m}^2$.

b. Hence, show that $4x^2 + 36x - 63 = 0$.

c. How wide is the pavement?