

# BC/HL Calculus

## 2016-2017 Summer Assignment

**Directions:** Complete this packet of review material.

- In order to be ready for your BC/HL Calculus Course you should be able to complete these prerequisite exercises. Be prepared to turn in your solutions and supporting work during the first week of school in September. Use additional paper if necessary.
- You may use the 2015 HL Calculus Formula Booklet provided as a separate document with this Summer Assignment.
- You may work with other students. However, you must be prepared to brief/explain your solutions using the supporting work on these pages in September.
- Your supporting work should include the neat, well-organized steps that lead to your solution.
- An exam will be given in early September to confirm your understanding of the pre-calculus concepts including the concept of a limit.
- Contact Mr. Acker at (908) 835-1973 or email (preferably) [racker@wmrhdsd.org](mailto:racker@wmrhdsd.org) if you have questions regarding the instructions.

Name: \_\_\_\_\_

## No Calculator Section

### 1. Sequences and Series

Find the value of  $k$  if  $\sum_{r=1}^{\infty} k \left(\frac{1}{3}\right)^r = 7$ .

### 2. Complex Numbers

If  $z_1 = a + a\sqrt{3}i$  and  $z_2 = 1 - i$ , where  $a$  is a real constant, express  $z_1$  and  $z_2$  in the form  $r \operatorname{cis} \theta$ , and hence find an expression for  $\left(\frac{z_1}{z_2}\right)^6$  in terms of  $a$  and  $i$ .

### 3. Complex Numbers

Given that  $z$  is the complex number  $x + iy$  and that  $|z| + z = 6 - 2i$ , find the value of  $x$  and the value of  $y$ .

#### 4. Trigonometry

Let  $f(x) = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$ .

- (a) For what values of  $x$  does  $f(x)$  not exist?
- (b) Simplify the expression  $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$ .

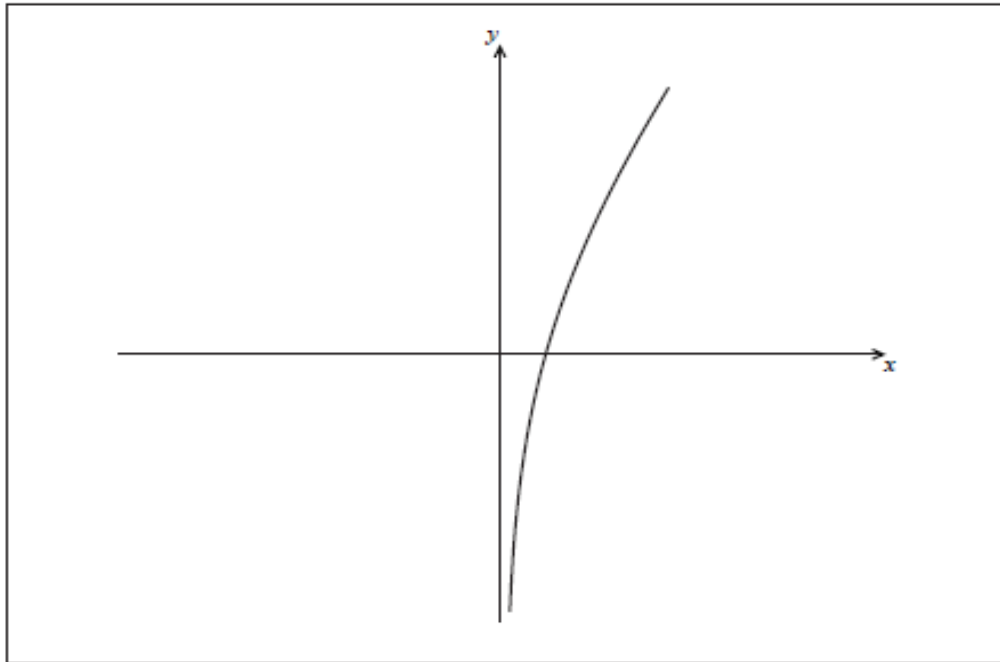
#### 5. Logarithms

Solve the equation  $2 - \log_3(x+7) = \log_{\frac{1}{3}} 2x$ .

## 6. Inverse Functions

The graph below shows  $y = f(x)$ , where  $f(x) = x + \ln x$ .

- (a) On the graph below, sketch the curve  $y = f^{-1}(x)$ .



- (b) Find the coordinates of the point of intersection of the graph of  $y = f(x)$  and the graph of  $y = f^{-1}(x)$ .

## 7. Trigonometry

In the triangle  $ABC$ ,  $\hat{A}BC = 90^\circ$ ,  $AC = \sqrt{2}$  and  $AB = BC + 1$ .

- (a) Show that  $\cos \hat{A} - \sin \hat{A} = \frac{1}{\sqrt{2}}$ .
- (b) By squaring both sides of the equation in part (a), solve the equation to find the angles in the triangle.
- (c) Apply Pythagoras' theorem in the triangle  $ABC$  to find  $BC$ , and hence show that  $\sin \hat{A} = \frac{\sqrt{6} - \sqrt{2}}{4}$ .
- (d) Hence, or otherwise, calculate the length of the perpendicular from  $B$  to  $[AC]$ .

## 8. Math Induction and Matrices

(a)  $A$  and  $U$  are square matrices, and  $X = U^{-1}AU$ . Use mathematical induction to prove that  $X^n = U^{-1}A^nU$ , for  $n \in \mathbb{Z}^+$ .

(b) Let  $A = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$  and  $U = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$ .

(i) Find the matrix  $D$  such that  $AU = UD$ .

(ii) Write down the matrix  $D^2$ .

(iii) Hence prove that  $A^{2n} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ , for  $n \in \mathbb{Z}^+$ .

(iv) Using the result from part (iii), show that  $(A^n)^{-1} = A^n$ , for  $n \in \mathbb{Z}^+$ .

## 9. Probability

A team of 6 players is to be selected from 10 volleyball players, of whom 8 are boys and 2 are girls.

- (a) In how many ways can the team be selected?
- (b) In how many of these selections is exactly one girl in the team?
- (c) If the selection of the team is made at random, find the probability that exactly one girl is in the team.

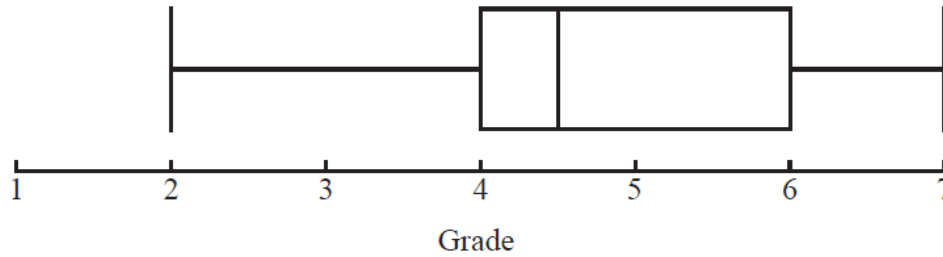
## Calculator Section 2012 HL Exam

### 10. Three Dimensional Lines and Planes

The planes  $2x + 3y - z = 5$  and  $x - y + 2z = k$  intersect in the line  $5x + 1 = 9 - 5y = -5z$ .  
Find the value of  $k$ .

## 11. Statistics

The box and whisker plot below illustrates the IB grades obtained by 100 students.



IB grades can only take integer values.

- (a) How many students obtained a grade of more than 4?
- (b) State, with reasons, the maximum possible number and minimum possible number of students who obtained a 4 in the exam.



## 12. Transformations

Let  $f(x) = \ln x$ . The graph of  $f$  is transformed into the graph of the function  $g$  by a translation of  $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ , followed by a reflection in the  $x$ -axis. Find an expression for  $g(x)$ , giving your answer as a single logarithm.

## 12. Binomial Expansion

Find the constant term in the expansion of  $\left(x - \frac{2}{x}\right)^4 \left(x^2 + \frac{2}{x}\right)^3$ .

### 13. Vectors

The coordinates of points A, B and C are given as  $(5, -2, 5)$ ,  $(5, 4, -1)$  and  $(-1, -2, -1)$  respectively.

- (a) Show that  $AB = AC$  and that  $\hat{BAC} = 60^\circ$ .
- (b) Find the Cartesian equation of  $\Pi$ , the plane passing through A, B, and C.
- (c)
  - (i) Find the Cartesian equation of  $\Pi_1$ , the plane perpendicular to  $(AB)$  passing through the midpoint of  $[AB]$ .
  - (ii) Find the Cartesian equation of  $\Pi_2$ , the plane perpendicular to  $(AC)$  passing through the midpoint of  $[AC]$ .
- (d) Find the vector equation of  $L$ , the line of intersection of  $\Pi_1$  and  $\Pi_2$ , and show that it is perpendicular to  $\Pi$ .

## 14. Limits

Go to website:

<https://www.math.ucdavis.edu/~kouba/CalcOneDIRECTORY/limcondirectory/LimitConstant.html>

Complete problems 1 through 13. Then check your solutions using the website. Be prepared to complete similar limit problems in September.